電磁波

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http://cobalt.cneas.tohoku.ac.jp/users/sato/newpage9.htm

電磁波と利用技術の移り変わり

- より高い周波数へ
- 使えるデバイス技術

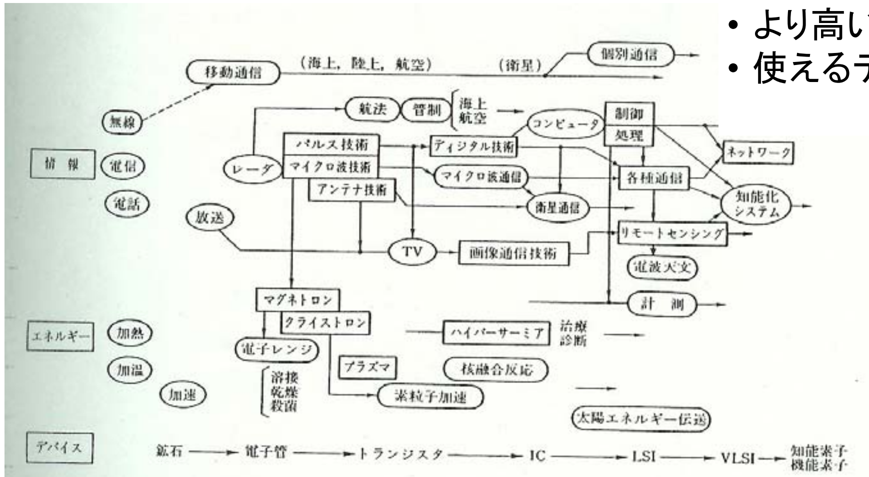


図 1.10　電波利用の主要技術と移り変り

八木宇田アンテナ

八木　秀次

宇田　新太郎

次世代携帯電話　5G

- 多周波数
- 多偏波
- Multi Input Multi Output :MIMO

ネットワークの高密度化のイメージ

Massive MIMO のイメージ

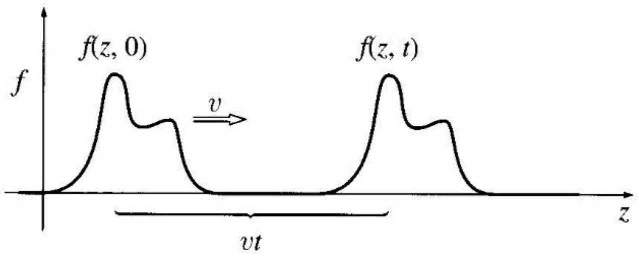
http://www.fujitsu.com/jp/group/mtc/technology/trend/g5-requirements/

Electromagnetic Waves

Introduction to Electrodynamics
David J. Griffiths
Fourth Edition

9.1 Waves in One Dimension

One Dimensional Wave



$$f(z,t)= f(z-vt,0)=g(z-vt) \qquad g(z)\equiv f(z,0)$$

波動は波形を変えずに一定速度で移動する。 拡散現象は波形が変化する。

2階線形微分方程式

- Wave equation
- Diffusion Equation

$$\Delta=\nabla^2=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+.....+\frac{\partial^2}{\partial x_n^2}$$ ラプラシアン

$$\boldsymbol{x}=(x_1,x_2,....,x_n)$$

$$u=u(\boldsymbol{x},t)=u(x_1,x_2,....,x_n,t)$$

$$\nabla^2 u(\boldsymbol{x})=0$$

(1)楕円形
Laplace の方程式

$$\nabla^2 u(\boldsymbol{x})=-\rho(\boldsymbol{x})$$

Poissonの方程式

$$\frac{\partial u(\boldsymbol{x},t)}{\partial t}=k\nabla^2 u+q(\boldsymbol{x},t)$$

(2)放物形
拡散方程式

$$\frac{\partial^2 u(\boldsymbol{x},t)}{\partial t^2}=v^2\nabla^2 u(\boldsymbol{x},t)$$

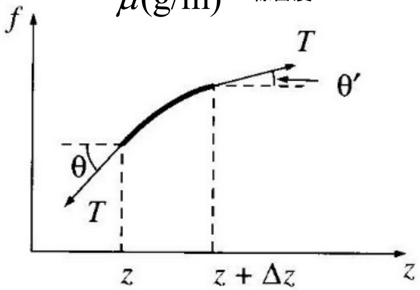
(3)双曲型
波動方程式

A stretched string

$f\,(\mathrm{m})$ 平衡点からの変位

$T\,(\mathrm{N})$ 張力

$\mu(\mathrm{g/m})$ 線密度



$$\Delta F = T\sin\theta' - T\sin\theta$$

$$\Delta F \simeq T\left(\tan\theta' - \tan\theta\right)$$

$$= T\left(\left.\frac{\partial f}{\partial z}\right|_{z+\Delta z} - \left.\frac{\partial f}{\partial z}\right|_{z}\right) \cong T\,\frac{\partial^2 f}{\partial z^2}\Delta z$$

$$\Delta F = \mu\left(\Delta z\right)\frac{\partial^2 f}{\partial t^2}$$

$$\frac{\partial^2 f}{\partial z^2} = \frac{\mu}{T}\frac{\partial^2 f}{\partial t^2}$$

弦の振動に関する運動方程式の導出から1次元波動方程式が導出できる

Wave equation （1次元波動方程式）

$$\frac{\partial^2 f}{\partial z^2} = \frac{\mu}{T}\frac{\partial^2 f}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

$v = \sqrt{\dfrac{T}{\mu}}$ 波動の伝搬速度が微分方程式から定まる

Solution

$$f(z,t) = g(z - vt)$$

$$f(z,t) = g(z + vt)$$

$$f(z,t) = g(z - vt) + h(z + vt)$$ 一般解

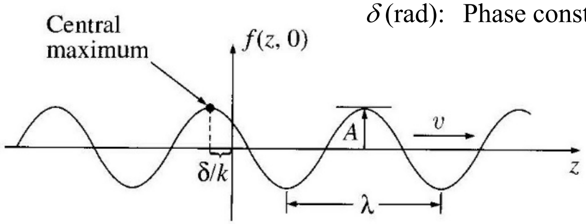
Sinusoidal wave 正弦波

A: Amplitude 振幅

k (rad/m): Wave number 波数

ω (rad/s): Angular frequency 角周波数

δ (rad): Phase constant 位相定数



$$kv = \omega$$

$$k = \frac{\omega}{v}$$

$$f(z,t) = A\cos\left[k(z - vt) + \delta\right]$$

$$f(z,t) = A\cos\left[kz - \omega t + \delta\right]$$

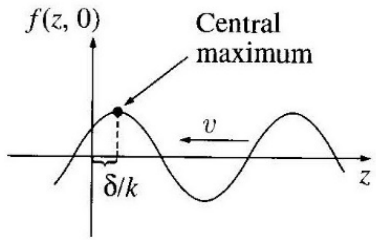
Terminology

$$f(z,t) = A\cos\left[kz + \omega t + \delta\right]$$

k (rad/m): Wave number 波数

ω (rad/s): Angular frequency 角周波数

δ (rad): Phase constant 位相定数



$$f(z,0) = A\cos\left[kz + \delta\right]$$

Complex notation （フェザー表示）

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$f(z,t) = \mathrm{Re}\left[A e^{i(kz-\omega t+\delta)} \right]$$

$$\tilde{f}(z,t) \equiv \tilde{A} e^{i(kz-\omega t)}$$

$$\tilde{A} \equiv A e^{i\delta}$$

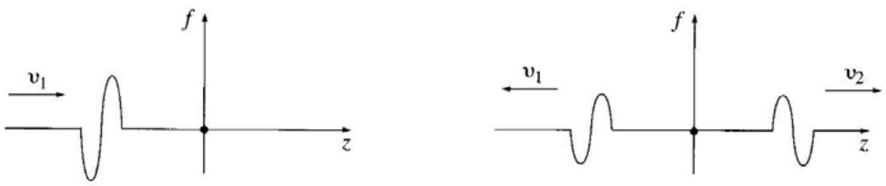
$$f(z,t) = \mathrm{Re}\left[\tilde{f}(z,t) \right]$$

Linear combination of sinusoidal waves

$$\tilde{f}(z,t) = \int_{-\infty}^{\infty} \tilde{A}(k) e^{i(kz-\omega t)} dk$$

正弦波は特殊な波形であるが、任意の波形は正弦波の重ね合わせで表現できる

Incident, Reflected and Transmitted Waves



(a) Incident pulse (b) Reflected and transmitted pulses

Plane Waves near a Boundary

$$\tilde{f}_I(z,t) = \tilde{A}_I e^{i(k_1 z-\omega t)}, (z<0)$$

Incident Wave 入射波

$$\tilde{f}_R(z,t) = \tilde{A}_R e^{i(-k_1 z-\omega t)}, (z<0)$$

Reflected Wave 反射波

$$\tilde{f}_T(z,t) = \tilde{A}_T e^{i(k_2 z-\omega t)}, (z>0)$$

Transmitted wave 透過波

$$\frac{\lambda_1}{\lambda_2} = \frac{k_2}{k_1} = \frac{v_1}{v_2}$$

Sinusoidal wave representation

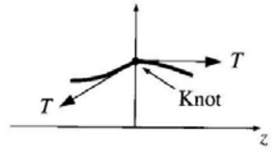
$$\tilde{f}(z,t)=\begin{cases}\tilde{A}_I e^{i(k_1 z-\omega t)}+\tilde{A}_R e^{i(-k_1 z-\omega t)} & \text{For } z<0\\ \tilde{A}_T e^{i(k_2 z-\omega t)} & \text{For } z>0\end{cases}$$

波動方程式（微分方程式）に一般解のうち、物理的に存在しえる解を書きくだしている。
振幅は未定とし、境界条件から定める。

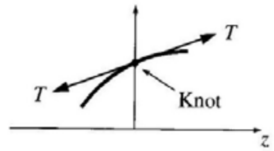
Boundary conditions　境界条件　初期条件

$$f(0^-,t)=f(0^+,t) \qquad \tilde{f}(0^-,t)=\tilde{f}(0^+,t)$$

$$\left.\frac{\partial f}{\partial t}\right|_{0^-}=\left.\frac{\partial f}{\partial t}\right|_{0^+} \qquad \left.\frac{\partial \tilde{f}}{\partial t}\right|_{0^-}=\left.\frac{\partial \tilde{f}}{\partial t}\right|_{0^+}$$



(a) Discontinuous slope; force on knot



(a) Continuous slope; no force on knot

Solutions

境界条件を一般解に適用して得られた連立方程式

$$\tilde{A}_I+\tilde{A}_R=\tilde{A}_T,\, k_1\left(\tilde{A}_I-\tilde{A}_R\right)=k_2\tilde{A}_T$$

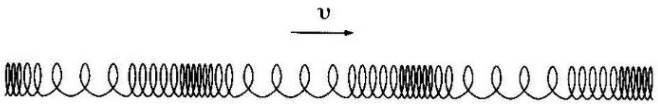
連立方程式を解くことで得られた解

$$\tilde{A}_R=\left(\frac{k_1-k_2}{k_1+k_2}\right)\tilde{A}_I \qquad \tilde{A}_T=\left(\frac{2k_1}{k_1+k_2}\right)\tilde{A}_I$$

$$\tilde{A}_R=\left(\frac{v_2-v_1}{v_2+v_1}\right)\tilde{A}_I \qquad \tilde{A}_T=\left(\frac{2v_2}{v_2+v_1}\right)\tilde{A}_I$$

Polarization （偏波）

Longitudinal polarization （縦波）



Transverse Wave （横波 横断面内の波）

$$\tilde{\boldsymbol{f}}_v(z,t) = \tilde{A}e^{i(kz-\omega t)}\hat{\boldsymbol{x}}$$ Horizontal Wave 水平偏波

$$\tilde{\boldsymbol{f}}_h(z,t) = \tilde{A}e^{i(kz-\omega t)}\hat{\boldsymbol{y}}$$ Vertical Wave 垂直偏波

$$\tilde{\boldsymbol{f}}\ (z,t) = \tilde{A}e^{i(kz-\omega t)}\hat{\boldsymbol{n}}$$

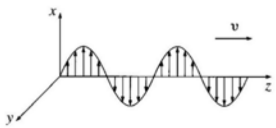
電磁界では電界の方向で偏波を規定する。

$\hat{\boldsymbol{n}}$ ：偏波方向ベクトル

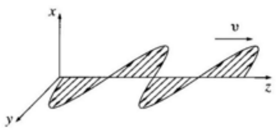
$$\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{z}} = 0$$

$$\hat{\boldsymbol{n}} = \cos\theta\hat{\boldsymbol{x}} + \sin\theta\hat{\boldsymbol{y}}$$

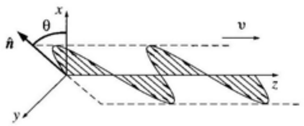
Transverse Waves



(a) Vertical polarization



(b) Horizontal polarization



(c) Polarization vector

$$\tilde{\boldsymbol{f}}\ (z,t) = \left(\tilde{A}\cos\theta\right)e^{i(kz-\omega t)}\hat{\boldsymbol{x}} + \left(\tilde{A}\sin\theta\right)e^{i(kz-\omega t)}\hat{\boldsymbol{y}}$$

9.2 Electromagnetic Waves in Vacuum

Maxwell's equation

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

Three-dimensional wave equation

$$\nabla \times \nabla \times \boldsymbol{E} = \nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} = \nabla \times \left(-\frac{\partial \boldsymbol{B}}{\partial t}\right)$$

$$= -\frac{\partial}{\partial t}(\nabla \times \boldsymbol{B}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\nabla \cdot \boldsymbol{E} = 0$$

$$\nabla^2 \boldsymbol{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} \qquad \nabla^2 \boldsymbol{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{B}}{\partial t^2}$$

One-Dimensional wave Equation

$$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 m/s \equiv c$$ 真空中の光速

波動方程式

$$\nabla^2 \boldsymbol{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0 \quad \nabla^2 \boldsymbol{H} - \mu_0 \varepsilon_0 \frac{\partial^2 \boldsymbol{H}}{\partial t^2} = 0$$

直交座標系では

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} = 0$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial t^2} = 0$$

平面波の導出

仮定

(1) : 波動は z方向に伝搬する

(2) : 波動はx-y平面内で一様である.

$$\frac{\partial^2 E_x}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_x}{\partial t^2} = 0$$

$$\frac{\partial^2 E_y}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2} = 0$$

$$\frac{\partial^2 E_z}{\partial z^2} - \mu_0 \varepsilon_0 \frac{\partial^2 E_z}{\partial t^2} = 0$$

Maxwellの方程式に戻る

$\nabla \cdot \boldsymbol{E} = 0$ に前記条件を入れて

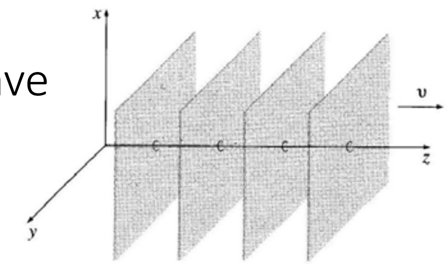
$$\nabla \cdot \boldsymbol{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\partial E_z}{\partial z} = 0$$

Z方向の変化があることを認めるためには $E_Z = 0$

つまり電界z成分は存在しない

電磁波は横波である

Monochromatic plane wave



$$\tilde{\boldsymbol{E}}(z,t) = \tilde{\boldsymbol{E}}_0 e^{i(kz-\omega t)}, \tilde{\boldsymbol{B}}(z,t) = \tilde{\boldsymbol{B}}_0 e^{i(kz-\omega t)}$$

$$\nabla \cdot \boldsymbol{E} = 0, \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\left(\tilde{E}_0\right)_z = \left(\tilde{B}_0\right)_z = 0$$

$$-k\left(\tilde{E}_0\right)_y = \omega\left(\tilde{B}_0\right)_x, k\left(\tilde{E}_0\right)_x = \omega\left(\tilde{B}_0\right)_y$$

$$\tilde{\boldsymbol{B}}_0 = \frac{k}{\omega}\left(\hat{\boldsymbol{z}} \times \tilde{\boldsymbol{E}}_0\right) \qquad B_0 = \frac{k}{\omega} E_0$$

$$\frac{k}{\omega} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c$$

$$\eta_0 \equiv \frac{\mu_0}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 120\pi$$

Intrinsic Impedance
固有インピーダンス

The electromagnetic spectrum

The Electromagnetic Spectrum

Frequency (Hz)	Type	Wavelength (m)
10^{22}		10^{-13}
10^{21}	gamma rays	10^{-12}
10^{20}		10^{-11}
10^{19}		10^{-10}
10^{18}	x rays	10^{-9}
10^{17}		10^{-8}
10^{16}	ultraviolet	10^{-7}
10^{15}	visible	10^{-6}
10^{14}	infrared	10^{-5}
10^{13}		10^{-4}
10^{12}		10^{-3}
10^{11}		10^{-2}
10^{10}	microwave	10^{-1}
10^{9}		1
10^{8}	TV, FM	10
10^{7}		10^{2}
10^{6}	AM	10^{3}
10^{5}		10^{4}
10^{4}	RF	10^{5}
10^{3}		10^{6}

The Visible Range

Frequency (Hz)	Color	Wavelength (m)
1.0×10^{15}	near ultraviolet	3.0×10^{-7}
7.5×10^{14}	shortest visible blue	4.0×10^{-7}
6.5×10^{14}	blue	4.6×10^{-7}
5.6×10^{14}	green	5.4×10^{-7}
5.1×10^{14}	yellow	5.9×10^{-7}
4.9×10^{14}	orange	6.1×10^{-7}
3.9×10^{14}	longest visible red	7.6×10^{-7}
3.0×10^{14}	near infrared	1.0×10^{-6}

電磁波とスペクトラム

Figure 3.1 Electromagnetic spectrum.

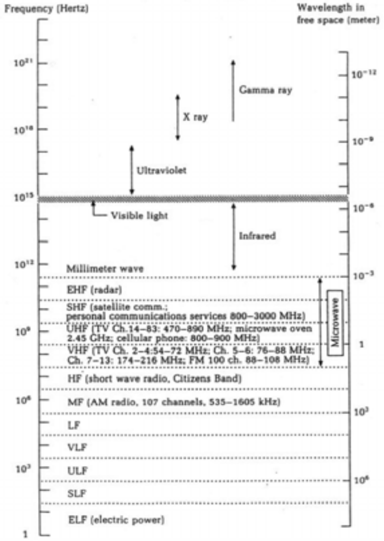
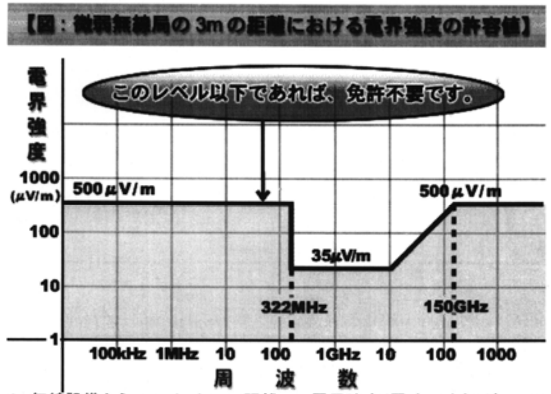


図3.5.1 電磁波の周波数スペクトル

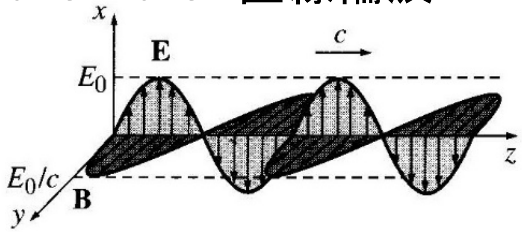
微弱無線局の規定 (電波法）

1. 無線設備から3メートルの距離での電界強度（電波の強さ）が、次の図に示されたレベルより低いものであれば、無線局の免許を受ける必要はありません。
周波数や用途など制限はありません。



2. 無線設備から500メートルの距離での電界強度（電波の強さ）が、200μV／m以下のもので、周波数などが郵政省告示で定められている無線遠隔操縦を行うラジコンやワイヤレスマイク用などのものは、無線局の免許を受ける必要はありません。

Linearly Polarized Plane Wave 直線偏波



$$\tilde{\boldsymbol{E}}(z,t)=\tilde{\boldsymbol{E}}_0 e^{i(kz-\omega t)}\hat{\boldsymbol{x}},\ \tilde{\boldsymbol{B}}(z,t)=\frac{1}{c}\tilde{\boldsymbol{E}}_0 e^{i(kz-\omega t)}\hat{\boldsymbol{y}}$$

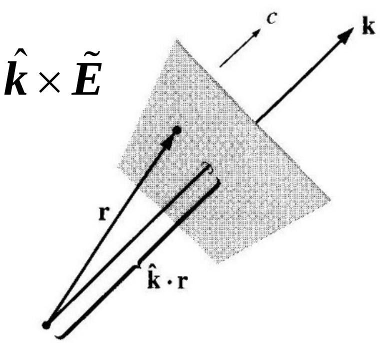
$$\boldsymbol{E}(z,t)=E_0\cos\left(kz-\omega t+\delta\right)\hat{\boldsymbol{x}},\ \boldsymbol{B}(z,t)=\frac{1}{c}E_0\cos\left(kz-\omega t+\delta\right)\hat{\boldsymbol{y}}$$

Propagation vector *k*

波数ベクトル：波数の大きさ、伝搬の方向をもつベクトル

$$\tilde{\boldsymbol{E}}(\boldsymbol{r},t)=\tilde{E}_0 e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\hat{\boldsymbol{n}}$$

$$\tilde{\boldsymbol{B}}(\boldsymbol{r},t)=\frac{1}{c}e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\left(\hat{\boldsymbol{k}}\times\hat{\mathbf{n}}\right)=\frac{1}{c}\hat{\boldsymbol{k}}\times\tilde{\boldsymbol{E}}$$



$$\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{k}}=0$$

エネルギー保存則

ローレンツ力を2個の荷電粒子に適用してエネルギー保存則から

$$-\frac{d}{dt}\left[\frac{1}{2}m_1\boldsymbol{v}_1^2+\frac{1}{2}m_2\boldsymbol{v}_2^2\right]=-\int_V\left[rot\boldsymbol{H}-\frac{\partial \boldsymbol{D}}{\partial t}\right]\cdot\boldsymbol{E}d^3x$$

更にファラデーの法則より

$$-\frac{d}{dt}\left[\frac{1}{2}m_1\boldsymbol{v}_1^2+\frac{1}{2}m_2\boldsymbol{v}_2^2\right]=-\int_V\left[\boldsymbol{H}\cdot\frac{\partial \boldsymbol{B}}{\partial t}+\boldsymbol{E}\cdot\frac{\partial \boldsymbol{D}}{\partial t}\right]d^3x+\int_V div(\boldsymbol{E}\times\boldsymbol{H})d^3x$$

$$=-\frac{d}{dt}\int_V\left[\frac{1}{2}\boldsymbol{H}\cdot\boldsymbol{B}+\frac{1}{2}\boldsymbol{E}\cdot\boldsymbol{D}\right]d^3x+\int_{S_0}(\boldsymbol{E}\times\boldsymbol{H})\cdot\mathbf{n}dS$$

ポインティングベクトル

これをまとめて

$$-\frac{d}{dt}\left[\frac{1}{2}m_1\boldsymbol{v}_1^2+\frac{1}{2}m_2\boldsymbol{v}_2^2+\frac{1}{2}\int_V[\boldsymbol{E}\cdot\boldsymbol{D}+\boldsymbol{B}\cdot\boldsymbol{H}]d^3x\right]=\int_{S_0}\boldsymbol{S}\cdot\mathbf{n}dS$$

$$\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}$$

Poynting Vector：ポインティングベクトル

エネルギー保存則

$$\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}$$

ポインティングベクトルは電界・磁界に垂直な断面を単位時間あたりに通過する電磁界のエネルギー流を表している。

また領域V内に蓄えられる電磁場のエネルギーは次式で与えられる。

$$U_{e.m.}=\frac{1}{2}\int_V[\boldsymbol{E}\cdot\boldsymbol{D}+\boldsymbol{B}\cdot\boldsymbol{H}]d^3x$$

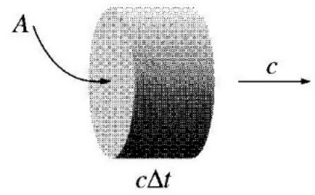
$$-\frac{d}{dt}\left[\frac{1}{2}m_1\boldsymbol{v}_1^2+\frac{1}{2}m_2\boldsymbol{v}_2^2+\frac{1}{2}\int_V[\boldsymbol{E}\cdot\boldsymbol{D}+\boldsymbol{B}\cdot\boldsymbol{H}]d^3x\right]=\int_{S_0}\boldsymbol{S}\cdot\boldsymbol{n}dS$$

Energy and Momentum in Electromagnetic Waves

$$u=\frac{1}{2}\left(\varepsilon_0E^2+\frac{1}{\mu_0}B^2\right)$$

$$B^2=\frac{1}{c^2}E^2=\mu_0\varepsilon_0E^2$$

$$u=\varepsilon_0E^2=\varepsilon_0E_0^2\cos^2(kz-\omega t+\delta)$$



$$\boldsymbol{S}=\frac{1}{\mu_0}(\boldsymbol{E}\times\boldsymbol{B})$$

Poynting vector : The energy flux density

$$\boldsymbol{S}=c\varepsilon_0E_0^2\cos^2(kz-\omega t+\delta)\hat{\boldsymbol{z}}=cu\hat{\boldsymbol{z}}$$

Monochromatic plane wave

2 Electromagnetic Waves in Vacuum

9.3 Electromagnetic Waves in Matter

Maxwell's Equation in Matter

$$\nabla \cdot \boldsymbol{E} = 0$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \times \boldsymbol{B} = \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

構成方程式

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$
$$\boldsymbol{H} = \frac{1}{\mu} \boldsymbol{B}$$

$$\nabla \cdot \boldsymbol{E} = 0$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \times \boldsymbol{B} = \mu \varepsilon \frac{\partial \boldsymbol{E}}{\partial t}$$

$$v = \frac{1}{\sqrt{\varepsilon \mu}} = \frac{c}{n}$$
$$n \equiv \sqrt{\frac{\varepsilon \mu}{\varepsilon_0 \mu_0}}$$

$$n \cong \sqrt{\varepsilon_r}$$ Index of refraction 屈折率

Poynting Vector in Matter ポインチングベクトル

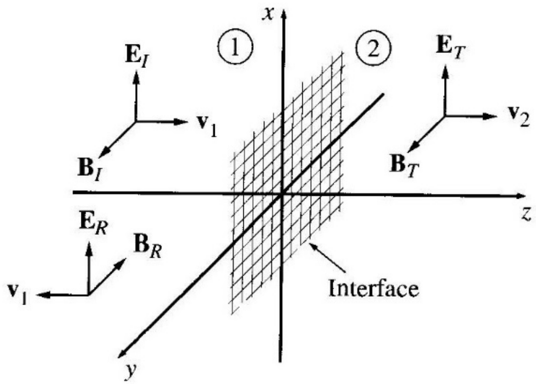
$$u = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu} B^2 \right)$$

$$\boldsymbol{S} = \frac{1}{\mu}(\boldsymbol{E} \times \boldsymbol{B}) = \boldsymbol{E} \times \boldsymbol{H}$$

Boundary conditions

$$\varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp}$$
$$B_1^{\perp} = B_2^{\perp}$$
$$\boldsymbol{E}_1^{\parallel} = \boldsymbol{E}_2^{\parallel}$$
$$\frac{1}{\mu_1} \boldsymbol{B}_1^{\parallel} = \frac{1}{\mu_2} \boldsymbol{B}_2^{\parallel}$$

Reflection at a Planer Boundary



Plane waves in Matter

$$\left.\begin{aligned}\tilde{\boldsymbol{E}}_I(z,t) &= \tilde{\boldsymbol{E}}_{0I}e^{i(k_1 z-\omega t)}\hat{\boldsymbol{x}}\\ \tilde{\boldsymbol{B}}_I(z,t) &= \frac{1}{v_1}\tilde{\boldsymbol{E}}_{0I}e^{i(k_1 z-\omega t)}\hat{\boldsymbol{y}}\end{aligned}\right\}$$

Incident wave

$$\left.\begin{aligned}\tilde{\boldsymbol{E}}_R(z,t) &= \tilde{\boldsymbol{E}}_{0R}e^{i(-k_1 z-\omega t)}\hat{\boldsymbol{x}}\\ \tilde{\boldsymbol{B}}_R(z,t) &= -\frac{1}{v_1}\tilde{\boldsymbol{E}}_{0I}e^{i(-k_1 z-\omega t)}\hat{\boldsymbol{y}}\end{aligned}\right\}$$

Reflected Wave

$$\left.\begin{aligned}\tilde{\boldsymbol{E}}_T(z,t) &= \tilde{\boldsymbol{E}}_{0T}e^{i(k_2 z-\omega t)}\hat{\boldsymbol{x}}\\ \tilde{\boldsymbol{B}}_T(z,t) &= \frac{1}{v_2}\tilde{\boldsymbol{E}}_{0T}e^{i(k_2 z-\omega t)}\hat{\boldsymbol{y}}\end{aligned}\right\}$$

Transmitted Wave

Boundary Conditions

$z = 0$

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$$

$$\frac{1}{\mu_1}\left(\frac{1}{v_1}\tilde{E}_{0I} - \frac{1}{v_1}\tilde{E}_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{v_2}\tilde{E}_{0T}\right)$$

$$\tilde{E}_{0I} - \tilde{E}_{0R} = \beta\tilde{E}_{0T}$$

$$\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$$

Solutions

$$\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I}, \tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}$$

$$\tilde{E}_{0R} = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\tilde{E}_{0I}, \tilde{E}_{0T} = \left(\frac{2v_2}{v_2 + v_1}\right)\tilde{E}_{0I}$$

Reflection and Transmission Coefficients

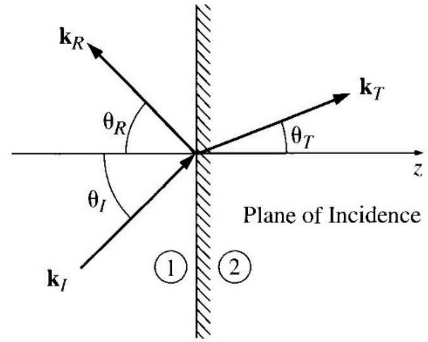
$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

$$T \equiv \frac{I_T}{I_I} = \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 = \frac{4 n_1 n_2}{(n_1 + n_2)^2}$$

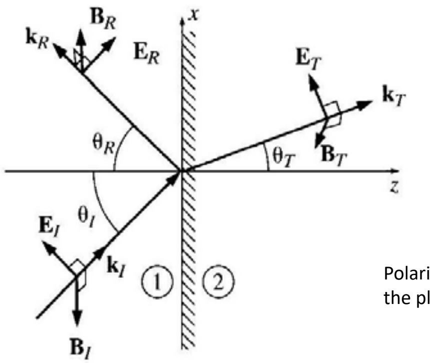
$$R + T = 1$$

エネルギー保存則 （垂直入射の場合）

Reflection and Transmission at Oblique Incidence

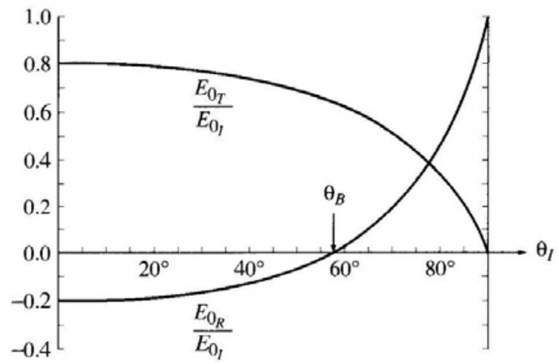


TM (Transverse Magnetic) mode

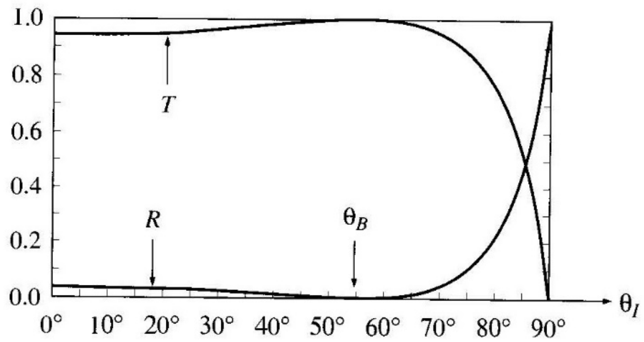


Polarization of incident wave is parallel to the plane of incidence

Fresnel’s reflection and Transmission Coefficients



Reflected and Transmitted Power



FD-TD

Finite-Difference Time-Domain

Maxwell方程式の　直接解法

電磁界散乱の数値解法

Maxwellの方程式 — FDTD法

波動方程式＋境界条件 — 波線追跡法

グリーン関数＋境界条件 — 積分法（モーメント法、境界要素法）

電磁界のエネルギー — 有限要素法(FEM)

Maxwellの方程式の一次元化

$$rot\mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} \qquad \frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial H_y}{\partial z}$$

$$rot\mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \qquad \frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\frac{\partial E_x}{\partial z}$$

時間ー空間の離散化

$$z = i \cdot \Delta z$$

$$t = n \cdot \Delta t$$

$$F^{(n)}(i) = F(z,t) = F(i \cdot \Delta z, n \cdot \Delta t)$$

空間の離散化と 電界・磁界のサンプル位置

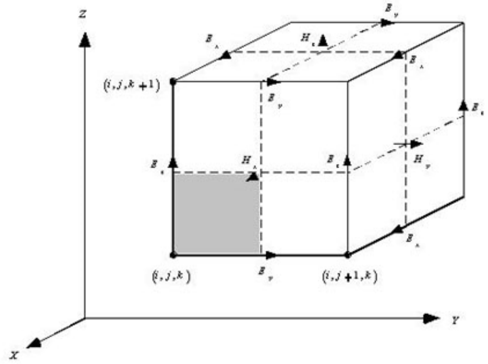


Figure 2-1: Yee's lattice for the FDTD method.

時間・空間微分の差分化

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial H_y}{\partial z}$$

$$\frac{E_x^{n+1}(i) - E_x^n(i)}{\Delta t} = -\frac{1}{\varepsilon(i)}\frac{H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i-\frac{1}{2})}{\Delta z}$$

$$E_x^{n+1}(i) = E_x^n(i) - \frac{\Delta t}{\Delta z \varepsilon(i)}\left[H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i-\frac{1}{2})\right]$$

Maxwellの方程式の差分化

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial H_y}{\partial z}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\frac{\partial E_x}{\partial z}$$

$$E_x^{n+1}(i) = E_x^n(i) - \frac{\Delta t}{\Delta z \varepsilon(i)}\left[H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i-\frac{1}{2})\right]$$

$$H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) = H_y^{n-\frac{1}{2}}(i+\frac{1}{2}) - \frac{\Delta t}{\Delta z \mu(i)}\left[E_x^n(i+1) - E_x^n(i)\right]$$

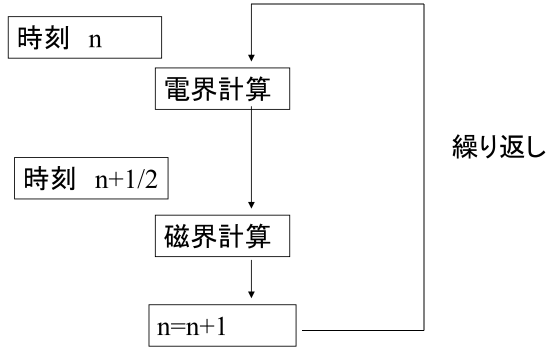
電磁界を計算のアルゴリズム

$$E_x^{n+1}(i) = E_x^n(i) - \frac{\Delta t}{\Delta z \varepsilon(i)} \left[H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) - H_y^{n+\frac{1}{2}}(i-\frac{1}{2}) \right]$$

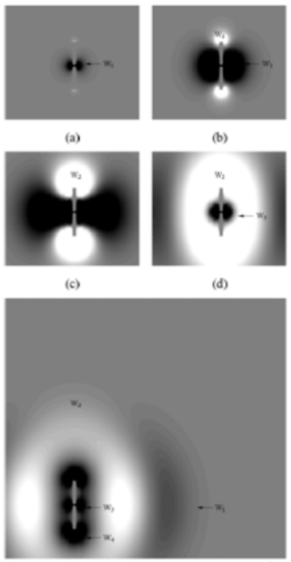
$$H_y^{n+\frac{1}{2}}(i+\frac{1}{2}) = H_y^{n-\frac{1}{2}}(i+\frac{1}{2}) - \frac{\Delta t}{\Delta z \mu(i)} \left[E_x^n(i+1) - E_x^n(i) \right]$$

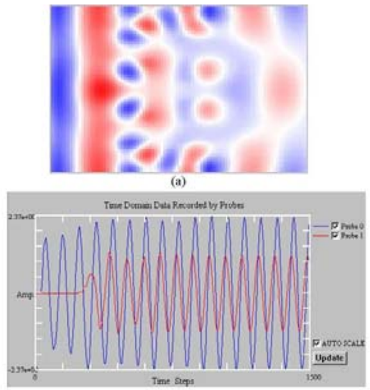
- 常に以前の時間における電磁界を使って未来の電磁界を計算する。
- 空間のパラメータ(導電率、誘電率)を容易に組み込める

FDTD計算アルゴリズム



FDTDシミュレーション

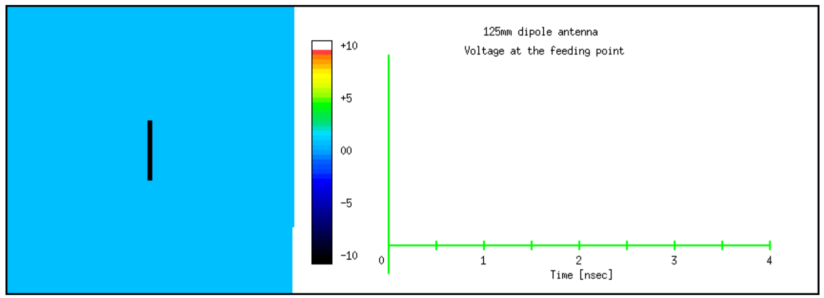

(a) (b) (c) (d) (e)



(a)
(b)

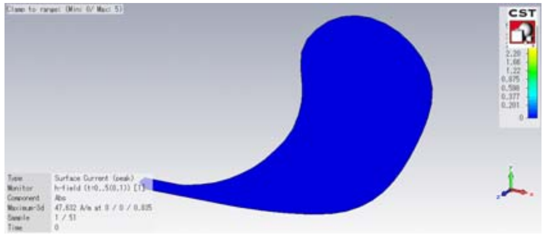
Figure 4-4: Field distribution (a) and time-domain data (b) with 11.14GHz sinusoidal excitation after 1500 time steps.

ダイポールアンテナからの放射

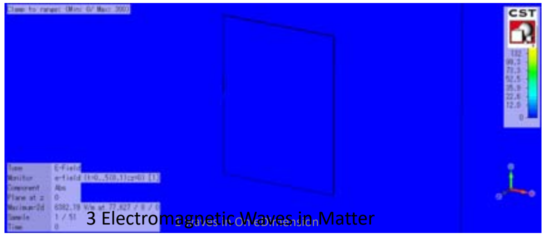


Surface current and E-field of Vivaldi antenna
calculated by CST MWS

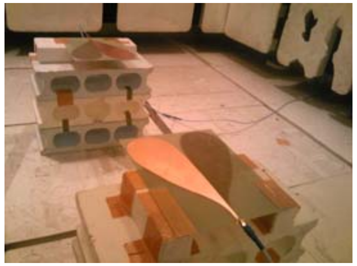
surface current

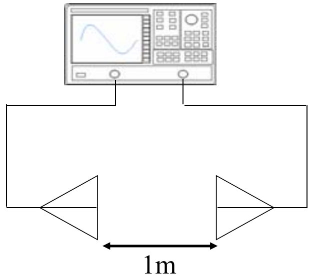


E-field



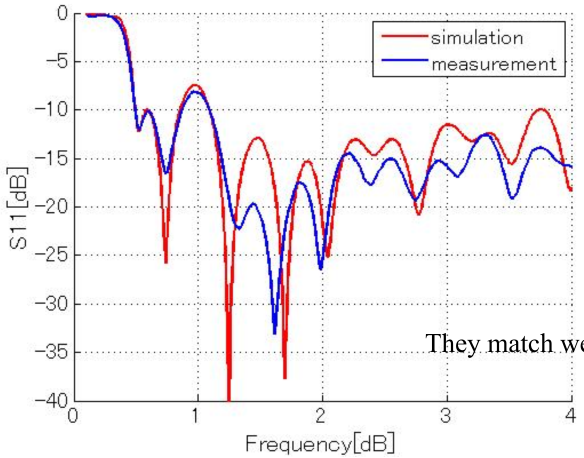
Comparison of antenna gain





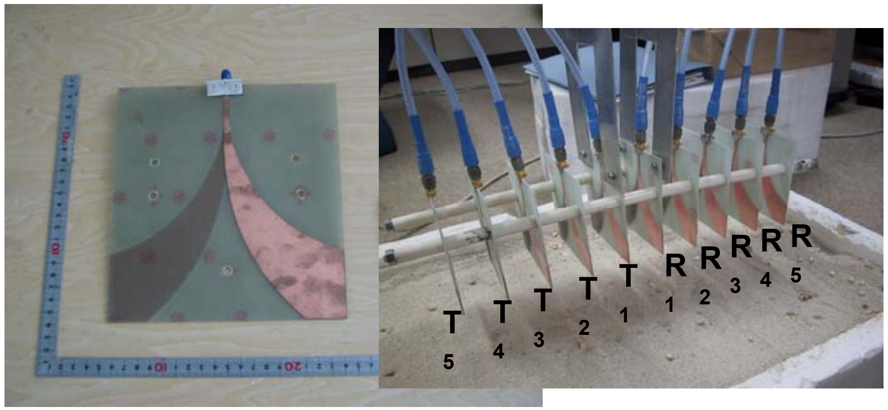
電波無響室 （機械・知能系共同棟）における実験

Return loss of the new Vivaldi antenna simulation vs. measurement

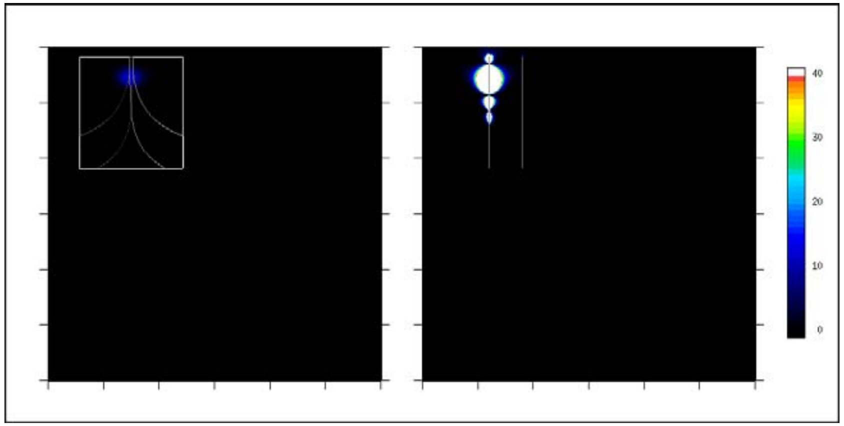


They match well enough!

SAR-GPR用に開発した広帯域アンテナアレイ



ビバルディアンテナから放射される電波の可視化



室

9.4 Absorption and Dispersion

吸収と分散

Maxwell's Equation in a Conducting Media

$$\boldsymbol{J}_f = \sigma \boldsymbol{E}$$

$$\nabla \cdot \boldsymbol{E} = \frac{1}{\varepsilon}\rho_f$$

$$\nabla \cdot \boldsymbol{B} = \boldsymbol{0}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{B} = \mu\sigma\boldsymbol{E} + \mu\varepsilon\frac{\partial \boldsymbol{E}}{\partial t}$$

Electromagnetic waves in Conducting Media

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu\sigma \frac{\partial \mathbf{E}}{\partial t}, \nabla^2 \mathbf{B} = \mu\varepsilon \frac{\partial^2 \mathbf{B}}{\partial t^2} + \mu\sigma \frac{\partial \mathbf{B}}{\partial t}$$

波動方程式＋拡散方程式

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(\tilde{k}z-\omega t)}, \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{i(\tilde{k}z-\omega t)}$$

$$\tilde{k}^2 = \mu\varepsilon\omega^2 + i\mu\sigma\omega$$

$$\tilde{k} = k + i\kappa$$

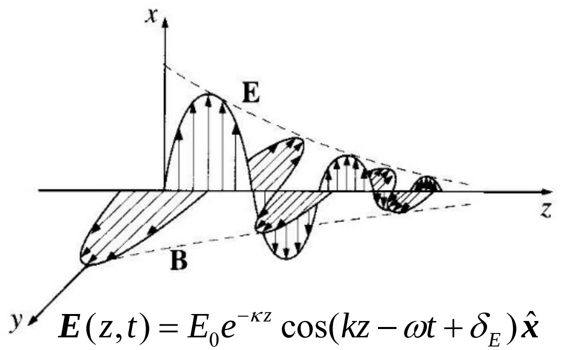
$$k = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}+1\right]^{1/2}, \kappa = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2}$$

位相定数、減衰定数

$$\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{-\kappa z} e^{i(kz-\omega t)}, \tilde{\mathbf{B}}(z,t) = \tilde{\mathbf{B}}_0 e^{-\kappa z} e^{i(kz-\omega t)}$$

$$d \equiv \frac{1}{\kappa}$$ Skin Depth

Plane Wave propagating in a Conducting Media



$$\mathbf{E}(z,t) = E_0 e^{-\kappa z}\cos(kz - \omega t + \delta_E)\hat{\mathbf{x}}$$

$$\mathbf{B}(z,t) = B_0 e^{-\kappa z}\cos(kz - \omega t + \delta_E + \phi)\hat{\mathbf{y}}$$

Boundary Conditions for a Conducting Media

$$\left.\begin{aligned} \varepsilon_1 E_1^{\perp} - \varepsilon_2 E_2^{\perp} = \sigma_f, \mathbf{E}_1^{\parallel} - \mathbf{E}_2^{\parallel} = 0 \\ B_1^{\perp} - B_2^{\perp} = 0, \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} - \frac{1}{\mu_2}\mathbf{B}_2^{\parallel} = \mathbf{K}_f \times \hat{\mathbf{n}} \end{aligned}\right\}$$

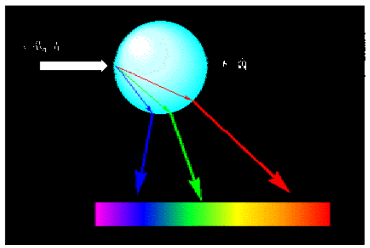
$\mathbf{K}_f$ Free surface current

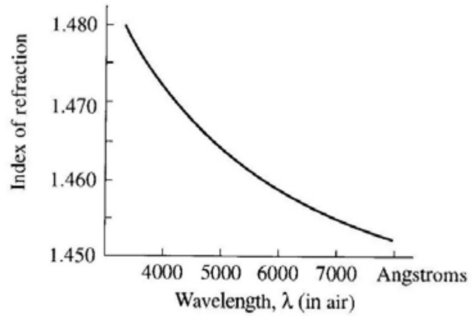
$$\tilde{\beta} \equiv \frac{\mu_1 v_1}{\mu_2 \omega}\tilde{k}_2$$

$$\tilde{E}_{0R} = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)\tilde{E}_{0I}, \tilde{E}_{0T} = \left(\frac{2}{1+\tilde{\beta}}\right)\tilde{E}_{0I}$$

$$\tilde{E}_{0R} = -\tilde{E}_{0I}, \tilde{E}_{0T} = 0$$ For a perfect conductor

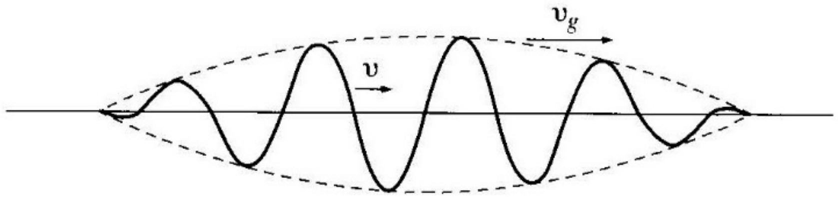
Index of Refraction of a Typical Glass





Firentze March 2019

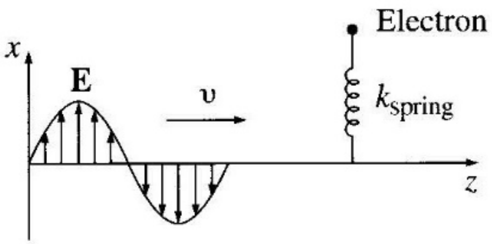
Phase velocity and Group velocity （群速度）



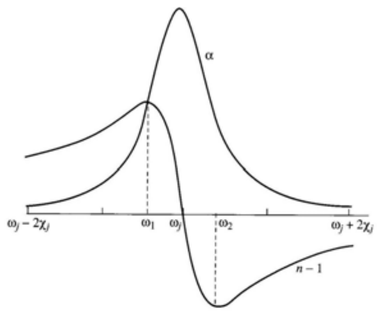
$v = \frac{\omega}{k}$ Phase velocity 位相速度

$v_g = \frac{d\omega}{dk}$ Group velocity 群速度

Model of electron



The index of refraction and the absorption coefficient

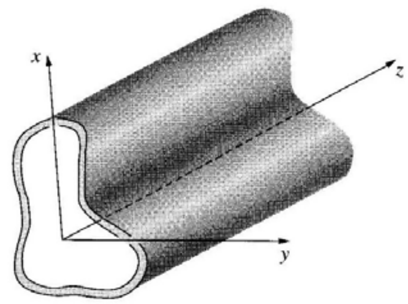


9.5 Guided Waves 導波路

Wave guide 導波管

$$\left.\begin{aligned} \boldsymbol{E}^{\parallel} &= 0 \\ B^{\perp} &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} \tilde{\boldsymbol{E}}(x,y,z,t) &= \tilde{\boldsymbol{E}}_0(x,y)e^{i(kz-\omega t)} \\ \tilde{\boldsymbol{B}}(x,y,z,t) &= \tilde{\boldsymbol{B}}_0(x,y)e^{i(kz-\omega t)} \end{aligned}\right\}$$



Electromagnetic Waves in a Wave Guide

$$\left.\begin{aligned} \tilde{\boldsymbol{E}}(x,y,z,t) &= \tilde{\boldsymbol{E}}_0(x,y)e^{i(kz-\omega t)} \\ \tilde{\boldsymbol{B}}(x,y,z,t) &= \tilde{\boldsymbol{B}}_0(x,y)e^{i(kz-\omega t)} \end{aligned}\right\}$$

$$\tilde{\boldsymbol{E}}_0 = E_x\hat{\boldsymbol{x}} + E_y\hat{\boldsymbol{y}} + E_z\hat{\boldsymbol{z}}$$

$$\tilde{\boldsymbol{B}}_0 = B_x\hat{\boldsymbol{x}} + B_y\hat{\boldsymbol{y}} + B_z\hat{\boldsymbol{z}}$$

$$\nabla\times\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla\times\boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z, \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -i\frac{\omega}{c^2}E_z$$

$$\frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x, \frac{\partial B_z}{\partial y} - ikB_y = -i\frac{\omega}{c^2}E_x$$

$$ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y, ikB_x - \frac{\partial B_z}{\partial x} = -i\frac{\omega}{c^2}E_y$$

Guided Waves

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega B_z, \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = -i\frac{\omega}{c^2}E_z$$

$$\frac{\partial E_z}{\partial y} - ikE_y = i\omega B_x, \frac{\partial B_z}{\partial y} - ikB_y = -i\frac{\omega}{c^2}E_x$$

$$ikE_x - \frac{\partial E_z}{\partial x} = i\omega B_y, ikB_x - \frac{\partial B_z}{\partial x} = -i\frac{\omega}{c^2}E_y$$

$$E_x = \frac{1}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial x} + \omega\frac{\partial B_z}{\partial y}\right)$$

$$E_y = \frac{1}{(\omega/c)^2 - k^2}\left(k\frac{\partial E_z}{\partial y} + \omega\frac{\partial B_z}{\partial x}\right)$$

$$B_x = \frac{1}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial x} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial y}\right)$$

$$B_y = \frac{1}{(\omega/c)^2 - k^2}\left(k\frac{\partial B_z}{\partial y} + \frac{\omega}{c^2}\frac{\partial E_z}{\partial x}\right)$$

Wave Equation of Guided Waves （固有方程式）

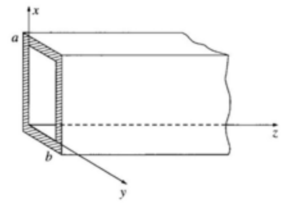
波数ベクトルは媒質だけでなく、導波管形状で定まる

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{\omega}{c}\right)^2-k^2\right]E_z=0$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{\omega}{c}\right)^2-k^2\right]B_z=0$$

B_z=0 TM waves
E_z=0 TE Waves
B_z=E_z=0 TEM Waves

Rectangular Wave Guide



$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{\omega}{c}\right)^2-k^2\right]B_z=0$$

$$B_z(x,y)=X(x)Y(y)$$

$$\frac{\partial^2 X}{\partial x^2}+X\frac{\partial^2 Y}{\partial y^2}+\left[\left(\frac{\omega}{c}\right)^2-k^2\right]XY=0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2}=-k_x^2,\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2}=-k_y^2,$$

$$-k_x^2-k_y^2+\left(\frac{\omega}{c}\right)^2-k^2=0$$

$$X(x)=A\sin\left(k_x x\right)+B\cos\left(k_y y\right)$$

$$k_x=m\pi/a(m=0,1,2,)$$

$$k_y=n\pi/b(m=0,1,2,)$$

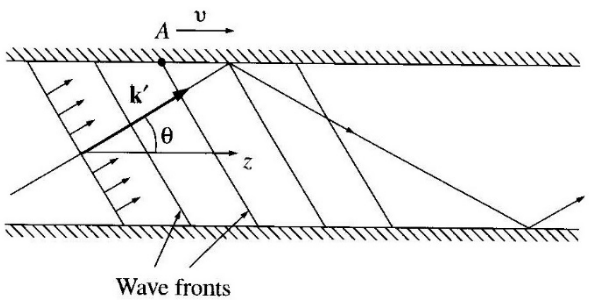
$$B_z=B_0\cos\left(m\pi x/a\right)\cos(n\pi y/b)$$

$$\omega<c\pi\sqrt{(\omega/c)^2-\pi^2\left[(m/a)^2+(n/b)^2\right]}\equiv\omega_{mn}$$

Cut-off frequency for Mode mn

波数ベクトルは離散的な値しか取れない

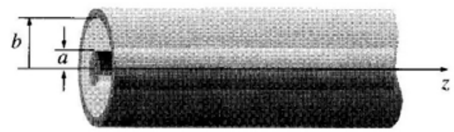
Plane Waves traveling in a Rectangular Wave Guide



Coaxial Cable 同軸ケーブル

Coaxial line admits a mode with

$$E_z=0,B_z=0$$



$$\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=i\omega B_z,\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=-i\frac{\omega}{c^2}E_z$$

$$\frac{\partial E_z}{\partial y}-ikE_y=i\omega B_x,\frac{\partial B_z}{\partial y}-ikB_y=-i\frac{\omega}{c^2}E_x$$

$$ikE_x-\frac{\partial E_z}{\partial x}=i\omega B_y,ikB_x-\frac{\partial B_z}{\partial x}=-i\frac{\omega}{c^2}E_y$$

$$k=\omega/c$$

$$cB_y=E_x,cB_x=-E_y$$

$$\frac{\partial E_x}{\partial x}-\frac{\partial E_y}{\partial y}=0,\frac{\partial E_y}{\partial x}-\frac{\partial E_x}{\partial y}=0$$

$$\frac{\partial B_x}{\partial x}-\frac{\partial B_y}{\partial y}=0,\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}=0$$

Waves in a Coaxial Line

$$\boldsymbol{E}(s,\phi,z,t) = \frac{A\cos(kz-\omega t)}{s}\hat{\boldsymbol{s}}$$

$$\boldsymbol{B}(s,\phi,z,t) = \frac{A\cos(kz-\omega t)}{cs}\hat{\phi}$$

TEMモード

- 同軸ケーブルで伝送する電磁波はTEMモード
- 絶縁された2つの導体があればTEMモードが伝搬可能
- 波数ベクトルは連続的
- 平面波と同じ振舞いをする